Nonequilibrium Magnetisation Reversal by Periodic Impulsive Fields in Ising Meanfield Dynamics

MUKTISH ACHARYYA
Department of Physics, Presidency College
86/1 College Street, Calcutta-700073, India
muktish.acharyya@gmail.com

We studied the *nonequilibrium* magnetisation reversal in kinetic Ising ferromagnets driven by periodic impulsive magnetic field in meanfield approximation. The meanfield differential equation was solved by sixth order Runge-Kutta-Felberg method. The periodicity and strength of the applied impulsive magnetic field play the key role in magnetisation reversal. We studied the minimum strength of impulsive field required for magnetisation reversal at any temperature as a function of the periodicity of the impulsive field. In the high temperature and small period this is observed to be linear. The results are compared with that obtained from Monte Carlo simulation of a three dimensional Ising ferromagnet.

Keywords: Ising model, Meanfield theory, Magnetisation reversal
PACS Nos: 75.60.Jk

I. Introduction:

The Ising model is a prototype to study some equilibrium as well as nonequilibrium phenomena [1]. Recently, the nonequilibrium aspects of Ising ferromagnets in presence of time varying fields become an interesting field of modern research [1]. The time dependent meanfield equation [2], of kinetic Ising ferromagnet in presence of applied magnetic field, is a simple representation of nonequilibrium phenomena. The nonequilibrium dynamic phase transition [1] is studied extensively in this system. It exhibits tricritical behaviour and a method of finding the tricritical points is proposed recently [3]. Like so called equilibrium transition, the nonequilibrium dynamic transition shows the divergence of time scale and power law divergence was found recently [4].

The magnetisation reversal, *by nucleation*, in the Ising ferromagnet was studied by Monte Carlo simulation (for large systems) in the presence of constant (in time) magnetic field [5]. This magnetisation reversal is an equilibrium phenomenon. Due to the presence of thermal fluctuation, any small amount of field (constant in time) may lead to the magnetisation reversal. In that sense no such minimum amount of field is required. However, in the meanfield approximation a minimum amount of field is required for magnetisation reversal. The externally applied periodic (in time) impulsive fields keeps the system always away from equilibrium. So, it would be interesting to know how the *nonequilibrium* magnetisation reversal occurs if one applies a periodic impulsive fields. In this paper, we addressed this question and studied the *nonequilibrium* magnetisation reversal (in kinetic Ising ferromagnets in meanfield approximation) in presence of periodic (in time) impulsive magnetic fields. The nonequilibrium magnetisation reversal was studied [6] recently in Ising model both by Monte Carlo simulation and by solving the dynamical meanfield equation. The dynamics of magnetisation reversal was also studied [7] experimentally in polycrystalline Co film.

We have arranged the paper as follows: the meanfield differential equation of kinetic Ising ferromagnet and the method of numerical solution are described in the next section. The numerical results are given in section-III. The paper end with a summary in section-IV.

II. Model and numerical solution:

The differential equation of instantaneous average magnetisation $m(t)$ of kinetic Ising ferromagnet, driven by a time varying magnetic field, in meanfield approximation, is given as [2]

$$\tau \frac{dm}{dt} = -m + \tanh(\frac{m + h(t)}{T}), \tag{1}$$

where, $h(t)$ is the externally applied time varying magnetic field and T is the temperature

measured in units of the Boltzmann constant (K_B). This equation describes the nonequilibrium behaviour of instantaneous value of magnetisation $m(t)$ of Ising ferromagnet in meanfield approximation. Here, τ stands for the microscopic relaxation time for the spin flip [2].

The periodic impulsive magnetic field is given as

$$\begin{aligned} h(t) &= -h_0, \text{for t} = \text{n}\Delta\text{t}, \text{where n} = 1, 2, 3. \\ &= 0 \quad \text{otherwise.} \end{aligned} \tag{2}$$

Where Δt and h_0 denote the periodicity and strength of the impulsive field respectively.

We have solved this equation by sixth order Runge-Kutta-Felberg (RKF) [8] method to get the instantaneous value of magnetisation $m(t)$ at any finite temperature T, h_0 and Δt. The method of solving the ordinary differential equation $\frac{dm}{dt} = F(t, m(t))$, by sixth order RKF method, is described briefly as:

$m(t+dt) = m(t) + \left(\frac{16k_1}{135} + \frac{6656k_3}{12825} + \frac{28561k_4}{56430} - \frac{9k_5}{50} + \frac{2k_6}{55}\right)$
where
$k_1 = dt \cdot F(t, m(t))$
$k_2 = dt \cdot F(t + \frac{dt}{4}, m + \frac{k_1}{4})$
$k_3 = dt \cdot F(t + \frac{3dt}{8}, m + \frac{3k_1}{32} + \frac{9k_2}{32})$
$k_4 = dt \cdot F(t + \frac{12dt}{13}, m + \frac{1932k_1}{2197} - \frac{7200k_2}{2197} + \frac{7296k_3}{2197})$
$k_5 = dt \cdot F(t + dt, m + \frac{439k_1}{216} - 8k_2 + \frac{3680k_3}{513} - \frac{845k_4}{4104})$
$k_6 = dt \cdot F(t + \frac{dt}{2}, m - \frac{8k_1}{27} + 2k_2 - \frac{3544k_3}{2565} + \frac{1859k_4}{4104} - \frac{11k_5}{40})$................................(3)

The time interval dt was measured in units of τ (the time taken to flip a single spin). Actually, we have used $dt = 0.01$ (setting τ=1.0). The local error involved [8] in the sixth order RKF method is of the order of $(dt)^6 (= 10^{-12})$. We started with initial condition $m(t = 0) = 1.0$.

III. Results:

Starting from the initial magnetisation $m(t = 0) = 1.0$, the instantaneous magnetisation ($m(t)$) decreases as time goes on. At any instant the magnetisation $m(t)$ becomes negative and this is known as magnetisation reversal. We studied this magnetisation reversal by applying the periodic impulsive field (Eqn.-2). The time required for magnetisation reversal depend on the value of Δt and h_0 and the temperature T and is called reversal time T_r. All times are measured in the unit of 0.01τ throughout this study. A typical results of magnetisation reversal is shown in Fig.-1, for $\Delta t = 10$ and $h_0 = 1.5$ at temperature $T = 0.90$. Here, the magnetisation reversal occurs at time $t = 760$. It

was observed that, at any temperature T, the reversal occurs early as we increase the strength h_0 keeping Δt fixed. In this case, a minimum value of strength, h^r_{min}, is required for reversal. We checked few cases, taking the value of h_0 less than h^r_{min} and observed no reversal even for $t = 5 \times 10^6$.

At any fixed temperature T, the minimum field strength (h^r_{min}) required for magnetisation reversal, decreases as the periodicity (Δt) decreases. Similarly, for fixed Δt, h^r_{min} decreases as the temperature increases. In the present study, we observed that, at temperature $T = 0.80$, for $\Delta t = 6$ and $\Delta t = 8$ the values of h^r_{min} becomes 0.33 and 0.43 respectively. Similarly, at temperature $T = 0.90$, for $\Delta t = 6$ and $\Delta t = 8$ the values of h^r_{min} become 0.13 and 0.17 respectively. It may be noted here that for static (in time) field the minimum values calculated for *equilibrium* magnetisation reversal are 0.07 and 0.03 for $T = 0.80$ and $T = 0.90$ respectively. At any particular temperature, the value of minimum field required (for periodic impulsive fields in nonequilibrium case) is always higher than that for the static field. These results are shown in Fig.-2. In this figure, the reversal time T_r is plotted against the field strength h_0 for different values of Δt and temperature T. The reversal time T_r is observed to increase here with the decrease of field strength h_0.

At any fixed temperature T, the minimum required field strength h^r_{min}, increases with the periodicity Δt of the impulsive fields. In the high temperature ($T_c = 1$ for equilibrium ferro-para transition) limit, it is observed that the relationship is linear (for small value of Δt). We have shown this in Fig.-3.

These results are compared with that obtained from Monte Carlo simulation. We have considered a three dimensional Ising model with nearest neighbour ferromagnetic interaction under periodic boundary condition in all directions. The Hamiltonian of this system is represented as

$$H = -J\Sigma_{<ij>}S_iS_j - h(t)\Sigma_i S_i \tag{3}$$

where $S_i = \pm 1$ is Ising spins, J is the nearest neighbour ferromagnetic interaction strength and $h(t)$ is periodic impulsive field (Eqn-2). We have considered a cubic lattice of size $L = 20$ (here). The initial condition was taken as all spins are up (i.e., $S_i = +1$ for all i). The updating scheme is taken as random. We select a spin randomly and calculated the energy in flipping, i.e., ΔE. We flipped the selected spin with probability $P_f = \mathrm{Min}[1, \exp(-\Delta E/K_BT)]$, following the Metropolis algorithm[9]. Here K_B is Boltzmann constant. The temperature T is measured in the unit of J/K_B. L^3 number of such random updating of Ising spins constitutes a single Monte Carlo step. This defines the unit of time in this simulation. In three dimensions the ferro-para transition temperature is $T_c = 4.511$ [9]. Now we allowed the system to follow the Metropolis dynamics in the presence of periodic impulsive field $h(t)$. At any fixed temperature T, we have calculated (averaged over 10 different random updating sequences) the minimum field required h^r_{min}

for the reversal of magnetisation and studied it as a function of the interval or periodicity Δt of periodic impulsive field. Here also, we have observed the linear variation of h^r_{min} with Δt. We have studied this for two different temperatures, here, $T = 4.0$ and $T = 4.2$. We observed that as the temperature increases the slope of the straight line (h^r_{min} versus Δt) decreases. This is shown in Fig-4. These Monte Carlo study supports the meanfield results.

IV. Summary:

We have studied the *nonequilibrium* magnetisation reversal of kinetic Ising ferromagnets in presence of periodic impulsive magnetic fields in meanfield approximation. At any finite temperature and fixed value of the periodicity of the impulsive field a minimum value of field strength is required for magnetisation reversal (sign change of instantaneous magnetisation). This value of minimum required field strength depends on the periodicity of the impulsive fields. As the periodicity increases the minimum field required increases. In the high temperature limit, it is observed to be linear in the periodicity (for small values) of the applied impulsive fields. This study differs from the case where the magnetic field is constant in time and consequently this leads to equilibrium magnetisation reversal. However, in this case since the field is time dependent, the characteristics of the magnetisation reversal is nonequilibrium type. These are studied in Ising meanfield dynamics. As a comparison the the nonequilibrium magnetisation reversal is also studied in three dimensional Ising ferromagnet driven by periodic impulsive fields by Monte Carlo simulation. The Monte Carlo results and the meanfield results are in good agreement.

Acknowledgments: The author would like to thank Urbashi Satpathi and Abhirup Patra for collecting few references. He would also like to thank Pradip Mukherjee for helping in typesetting of the manuscript.

References

1. B. K. Chakrabarti and M. Acharyya, *Rev. Mod. Phys.* **71**, 847 (1999); M. Acharyya, *Int. J. Mod. Phys C* **16**, 1631 (2005) and the references therein; M. Acharyya and B. K. Chakrabarti, *Annual Reviews of Computational Physics*, Vol. I, ed. D. Stauffer (World Scientific, Singapore, 1994), p. 107.

2. T. Tome and M. J. de Oliveira, *Phys. Rev. A* **41**, 4251 (1990).

3. M. Acharyya and A. B. Acharyya, *Comm. Comp. Phys.*, **3**, 397 (2008).

4. M. Acharyya and A. B. Acharyya, *Int. J. Mod. Phys.*, **21** 481 (2010).

5. M. Acharyya and D. Stauffer, *European Physical Journal* **B**, **5** 571 (1998); and the references therein.

6. A. Mishra and B. K. Chakrabarti, *Europhys. Lett.* **52** 311 (2000); A. Mishra and B. K. Chakrabarti, *J. Phys. A: Math and Gen* **33** 4249 (2000); See also, A. Chatterjee and B. K. Chakrabarti, *Phase transitions*, **77** 581 (2004).

7. H. Lai, Z. Huang, P. Gai, S. Chen and Y. Du, *Materials Science Forum*, **475** 2263 (2005).

8. C. F. Gerald and P. O. Wheatley, *Applied Numerical Analysis*, Pearson Education, (2006); See also, J. B. Scarborough, *Numerical Mathematical Analysis*, Oxford and IBH, (1930).

9. K. Binder and D. W. Heermann, *Monte Carlo Simulation in Statistical Physics,* Springer series in Solid State Sciences (springer, New York, 1997); D. P. Landau and K. Binder, *A guide to Monte Carlo Simulations in Statistical Physics* (Cambridge University Press, Cambridge, 2000).

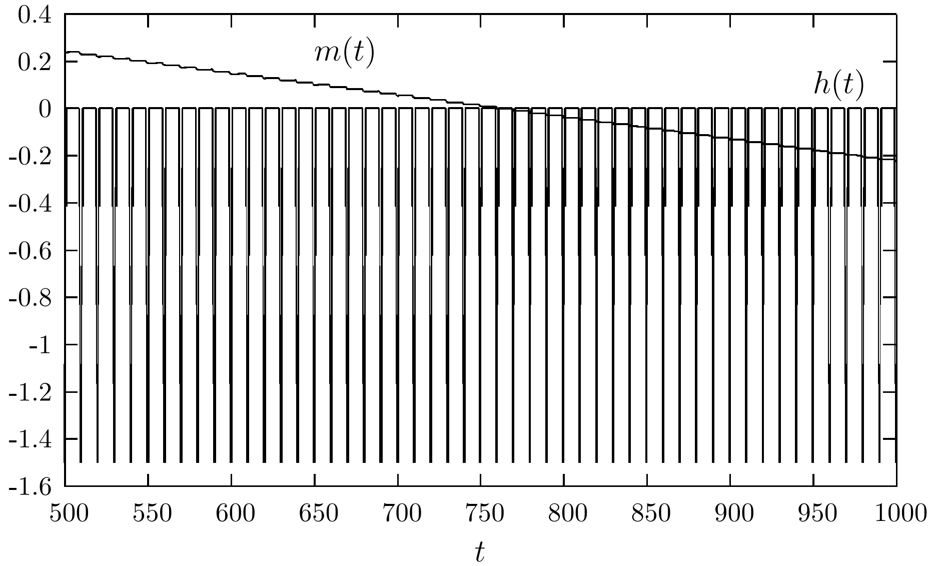

Fig-1. The time variation of magnetisation $m(t)$ and the applied periodic impulsive field $h(t)$ of periodicity $\Delta t = 10$ and strength $h_0 = 1.5$. The magnetisation reversal happens at $t = 760$. Here, the temperature $T = 0.90$.

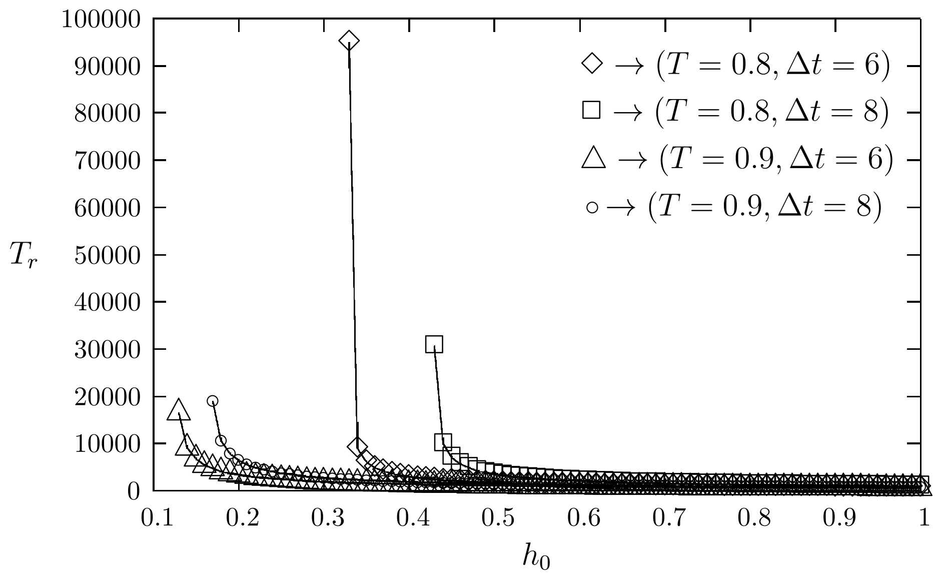

Fig-2. The variation of magnetisation reversal time (T_r) with respect to the strength of impulsive field (h_0) for different temperatures (T) and periodicity (Δt) of the impulsive fields.

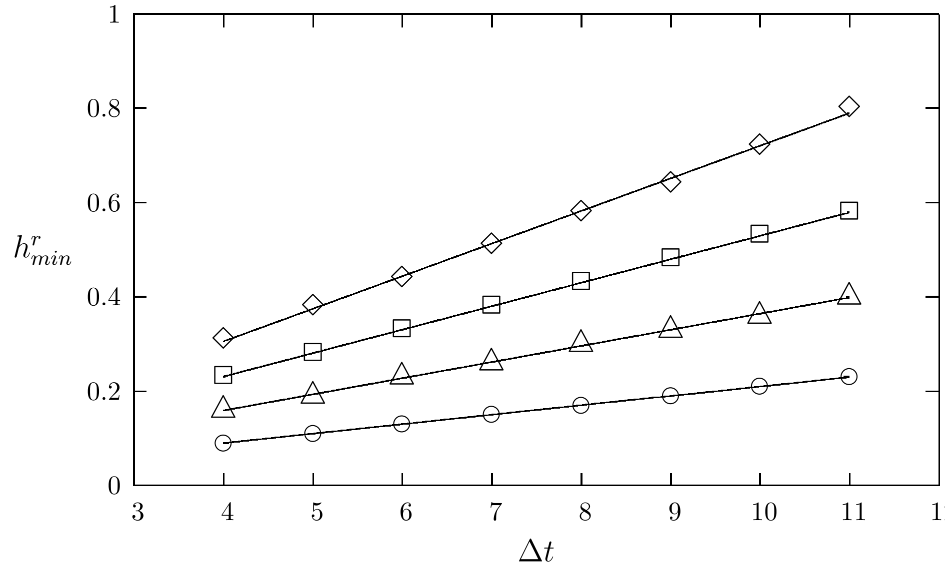

Fig-3. The variation of h^r_{min} with Δt for different temperatures (T) represented by different symbols. $T = 0.75(\diamond)$, $T = 0.80(\square)$, $T = 0.85(\triangle)$ and $T = 0.90(\mathrm{O})$. Continuous straight lines represent the linear best fit.

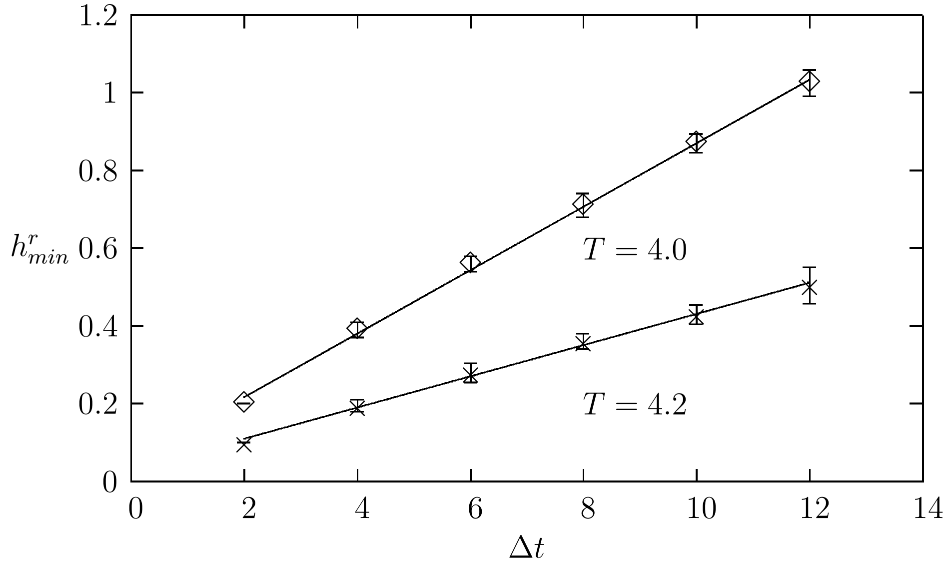

Fig.-4. The plots of h^r_{min} versus Δt for two different temperatures, obtained from Monte Carlo simulation. The errorbars in each data are shown by small vertical lines. The continuous straight line represents the linear best fit.